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Decoherence due to internal mesoscopic environment: a possible experimental test

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Abstract. Following the fundamentals of the Stern-Gerlach experiment, we propose for an experimental situation eventually revealing the decoherence effect due to the internal mesoscopic environment. The experiment-set-up we propose is a straightforward extension of the set-up recently used in the neutron optics interference experiments. First, we point to and discuss the occurrence of decoherence for the atom's path in the Stern-Gerlach experiment. Then, comparing a Stern-Gerlach apparatus with the apparatus of our set-up-proposal, we point out the occurrence of decoherence and consequently of non-violation of the Bell's inequality for a single atom's degree of freedom due to the environment consisting of the order of 10^2 particles.

PACS. 03.65.Yz Decoherence; open systems; quantum statistical methods – 03.75.Dg Atom and neutron interferometry – 03.65.Ud Entanglement and quantum nonlocality (e.g. EPR paradox, Bell's inequalities, GHZ states, etc.)

1 Introduction

The issue of decoherence is a topic of interest for a variety of the different areas of modern theoretical and applied physics. From the fundamental physical point of view, the decoherence effect is at the heart of the long-standing problem of the "transition from quantum to classical" [1,2]. In this context, it is usually invoked a plausible statement that, in the composite system "(open) system + environment $(S+E)$", the "environment (E)" should be a *sufficiently macroscopic* system in order to provide effective the decoherence process [1,2]. As a corollary to this conjecture naturally arises the expectation that a *mesoscopic environment* might be of only a weak practical use for the occurrence of the decoherence effect. Unfortunately, the list of the experiments on decoherence performed to date (cf., e.g., [3–5]) is neither long nor decisive *in this regard*.

On the other side, however, there are some still well-defined physical situations for which it is not easy (if possible at all) to recognize a macroscopic environment. E.g., in the standard Stern-Gerlach experiment, *there is not a room for a macroscopic "apparatus" (environment)*, thus challenging the above distinguished point of view. Furthermore, the standard model of the Stern-Gerlach (henceforth: SG) experiment can be straightforwardly extended to account for a truly *mesoscopic* (yet *internal*) environment. As a corollary to this extension, one concludes that in certain physical situations, a mesoscopic system can effectively play the role of the "environment" as defined in the foundations of the decoherence theory. With this observation under our belts, we are able to propose for an experimental situation for testing the occurrence of decoherence due to a mesoscopic yet internal environment. In a sense, with this, we indirectly test existence of a mesoscopic environment in the standard Stern-Gerlach experiment.

In this paper we propose for an experiment on the occurrence of the decoherence effect where a mesoscopic system consisting of the order of 10^2 particles acts effectively as the environment for the spin-1/2 degree of freedom. Our proposal can be recognized as a straightforward extension of the interference set-up recently used for the neutron interferometry as presented in reference [6]. The only change we suggest in the *zeroth approximation* of our proposal is the substitution of the neutron beam with the, e.g., Ag atom beam. Therefore, we propose for a situation, which compared to SG experiment, leads to a striking effect: the occurrence of decoherence and consequently non-violation of the Bell's inequality for the (ensemble of) *single atom*.

Operationally, the (non)occurrence of the decoherence effect can be tested indirectly, through testing validity of the Bell's inequality [7], in the manner exactly as it is performed in the recent experiment in the neutron optics interference experiment [6]. Certainly, non-violation of the Bell's inequality reveals the occurrence of the decoherence

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effect, while the violation of the Bell's inequality stems non-occurrence of the decoherence effect.

The contents of this paper are as follows. In Section 2, we give a compilation from the decoherence theory that justifies existence and the mesoscopic character of the internal environment in the standard SG experiment. Actually, our analysis of SG experiment reveals the occurrence of the decoherence effect *before* the atom reach the screen (i.e. before the atom's position-detection). The decoherence is due to the internal (mesoscopic) environment (R in our notation), which is usually *not* recognized and/or accounted for in the standard analysis of SG experiment; to this end, we emphasize the substantial role of the *sufficiently strong* external magnetic field of the SG-magnet, while the physical role of the mesoscopic environment R is carefully explained mainly in Appendices A and B. In Section 3, we point out the changes necessary to be made in the experimental set-up for the neutron optics experiment [6], thus defining a new physical situation of interest, cf. Figure 1 below. Now, *comparing a SG-apparatus with a part of the new set-up*, one concludes about the occurrence of the decoherence effect and consequently of non-violation of the Bell's inequality in the situation considered. In order to make the experimental results conclusive, we give the constraints on the set-up-parameters. In Section 4 we discuss our proposal in the light of the present state of art in the field of the Stern-Gerlach effect. Section 5 is discussion revealing the general physical relevance and importance of the experiment proposed. Section 6 is conclusion.

2 Internal mesoscopic environment in the Stern-Gerlach experiment

Following the fundamentals of the quantum measurement and the decoherence theory, in this section we distinguish the fact eventually not widely recognized or admitted, that in the standard Stern-Gerlach experiments the center-of-mass coordinates of the, e.g., Ag atom, plays the role of the "apparatus" as defined in the quantum measurement theory. This observation will prove to be essential for the arguments of the next section.

The composite system in the Stern-Gerlach (henceforth: SG) experiment is as follows: "atom's-spin + the spatial degrees of freedom of the atom + SG magnet + the screen". In this widely used and analyzed model, the SG-magnet field plays the role of the "external field" defining the external potential energy for the atom traversing the field. Therefore, in this (*generally used*) model of the experiment, *there is not a macroscopic system that might serve as the "apparatus" for a quantum measurement.* In the mathematical terms, the potential energy $\hat{V}$ is the "one-particle" observable depending on the degrees of freedom of the atom, while the external field appears as a classical variable (parameter) *not allowing the back-action of the atom to the field*; i.e. $\hat{V} = V(\hat{x}_i, \hat{p}_i, \vec{B}(\hat{x}_i))$, where $\vec{B}(\hat{x}_i)$ represents the external, *classical* magnetic field.

The net effect of the experiment (measurement of the spin-1/2) is the appearance of the two clear spots of the Ag atoms detected on the screen (detection of the atom's path before the screen would reveal the spin projection). But this is really the fundamental fact for our discussion. Actually, as the fundamentals of the quantum measurement theory [8,9] stem, the *non-appearance of the interference fringes on the screen* reveals the special role of the screen: the screen serves, here, as an "apparatus" of the quantum measurement of the so-called "second kind" [9], revealing the state of the object of measurement (here: of the atom) *before the detection*. In other words, the result of the measurement (position-detection on the screen) reveals that, *in front of the screen, the center-of-mass of the incoming atom has the well-defined semi-classical paths.* In the information-theoretic terms: before the screen, the Ag-atom's path is well-defined a classical information about the atom.

The point strongly to be emphasized is that the fundamentals of the decoherence theory [1,2,10,11] suggest that existence of the definite path (the definite classical information about the atom's path) can be established in the universally valid quantum mechanics *only through the occurrence of the decoherence effect* in the system "spin + the spatial degrees of freedom + SG magnet". But, now, being the external-field-source, the SG magnet *cannot play the role of the apparatus (environment)*. Therefore, one must, in the given model, refer to the following composite system: "spin + the spatial degrees of freedom".

This reasoning and the dilemma about the mesoscopic character of the composite system are neither new nor original. However, the decoherence theory offers the basis for justifying the center-of-mass of the atom as the effective "apparatus" in the SG experiment. Actually, the "atom" (i.e. its spatial degrees of freedom) is a subsystem of interest that can be "decomposed" in the different ways through the appropriate (linear) canonical transformations of the basic set of the variables (coordinates and momenta). Certainly, of interest are those transformations introducing the standard composite system "center-of-mass + relative coordinates (CM+R)" that can be directly connected to the SG experiment. [Actually, the original set of the coordinates, $\{x_{\alpha i}\}$, can be transformed to define the two sets of coordinates, $\{X_{CMi}; \xi_{Ri}^{(\alpha\beta)}\}$, and the CM system is defined by the set of the coordinates $X_{CMi} = \sum_{\alpha=1}^{N} x_{\alpha i} m_\alpha / \sum_{\alpha=1}^{N} m_\alpha$, N — the number of the particles in the system, $x_{\alpha i} = \vec{r}_\alpha \cdot \vec{e}_i$, $\vec{r}_\alpha$ is the position-variable of the αth particle, while the system R is defined by the "relative" coordinates $\xi_{Ri}^{(\alpha\beta)} = x_{\alpha i} - x_{\beta i}$, $\alpha, \beta = 1, 2, ...N$.] Namely, in the SG experiment, one indirectly observes the *definite paths of the CM system*, and the relative-coordinates system, R, remains properly to be included. To this end, and this is the point to be emphasized, the "relative coordinates system (R)" can be introduced as the CM's environment, i.e. as a "generalized apparatus" [10] performing the effective measurement of the atom's path. Yet, and as to the model considered, this is truly a *mesoscopic, internal environment* — which is the issue yet to be fully investigated both theoretically and experimentally in the general context of the decoherence theory, in which the system $S + CM + R$ can be recognized as the "object (of measurement) + apparatus + environment" [10], respectively.

Formally, the composite system of interest (*a single atom*) is defined by the following factorization of the Hilbert state space: $H_S \otimes H_{CM} \otimes H_R$, which bears obvious notation. Formal occurrence of the decoherence effect in this model is a straightforward extension of the existing and widely used model (cf., e.g., Bohm [12]), and the details are given in Appendix A. Actually, in Appendix A we show that entanglement-formation of the following type occurs in the system:

$$|\Psi\rangle_S|0\rangle_{CM}|0\rangle_R \to \sum_i C_i|i\rangle_S|i\rangle_{CM}|0\rangle_R \to \sum_i C_i|i\rangle_S|i\rangle_{CM}|i\rangle_R, \quad (1)$$

where $|\Psi\rangle_S = \sum_i C_i|i\rangle_S$ is the initial state of the (sub)system S. Then, the tracing out the "environment" R leads to the (improper [9]) mixed state for the composite system $S + CM$:

$$\hat{\rho}_{S+CM} = \sum_i |C_i|^2|i\rangle_{SS}\langle i| \otimes |i\rangle_{CMCM}\langle i|, \quad (2)$$

in the time interval of the order of the so-called *"decoherence time"*, τ_D [1,2]. Certainly, the environment R acts on the open system CM very much like the "apparatus" in quantum measurement acts on the object of measurement [10]; i.e. without the interaction in the system $CM + R$, the composite system $S + CM$ is subject to the Schrödinger law, which preserves the entanglement $\sum_i C_i|i\rangle_S|i\rangle_{CM}$.

From equation (2) it is obvious that the two subsystems, S and CM, bear the classical correlations of their states, as distinct from the "pure state" — entanglement as given in equation (1). This effect is observable at least for the time intervals of the order of τ, which is briefly discussed in Appendix B.

Prima facie, the decoherence equation (2) stems existence of a well-defined path for an atom in free space, i.e. in a situation that *no external field is applied*. However, this is another subtlety of the model discussed briefly in Appendix B. Therein, we point out nonexistence of the definite path in this situation which is the basis for the spatial interference of the atom's paths in the interference set-up presented in Figure 1, below. In Appendix B we also strongly emphasize the role of the external magnetic field $\vec{B}$ for both, SG experiment, and for the experiment herewith proposed. E.g., *only* for the sufficiently strong magnetic field one may justify the dynamics equation (1), which is substantial also for the experimental situation we are concerned with.

In the more elaborate terms, in Appendices A and B, we show that, for sufficiently strong magnetic field $\vec{B}$, the dynamics equations (1, 2) results as an interplay between the dynamical processes of establishing the entanglement in $S + CM$ system (the characteristic time τ_1) and the entanglement-establishing in the system $CM + R$ (the characteristic time τ_2), while the "decoherence time", $\tau_D = \tau_2 + \tau_3$, characterizes the decoherence effect equation (2), which is effective *before* the atom hits the screen. If the magnetic field is not sufficiently strong, then the dynamics equations (1, 2), interferes with the "wave packet spread" characterized by the time interval τ. Therefore, the desired effect might be observable in the time intervals shorter than this interval, τ.

So, we can conclude this Section with the following observations: (i) in the SG experiment, the "relative particle" degrees of freedom of the Ag atom plays effectively the role of the *internal* environment [13] acting as a quantum "apparatus" on the center-of-mass system, and (ii) the effect of this action can in principle be observed.

3 Quantum entanglement violation

We propose for an experiment to be performed in the manner essentially of reference [6], but with the *neutron beam substituted by the, say, Ag-atom beam*, cf. Figure 1. Following the conclusions of the preceding section, we are able to show in this section that this substitution might lead to non-violation of the Bell's inequality — as a consequence of the decoherence due to the *mesoscopic internal environment*, R.

For the experimental set-up, Figure 1, the pure entangled state of the type equation (1) occurs due to the interaction $\hat{H}_{int} = \mu_B \hat{S}_{Sx} \otimes B_x(\hat{x}_{CM})$, $\mu_B \equiv |e|\hbar/2mc$, and where the magnetic field $\vec{B}$ is the spin-turner (Mu-metal) magnetic field, while for simplicity we assume that $B_x = B'_\circ x$, and $\vec{B} = (B_x, 0, 0)$; the choice of the x-axis defines the two regions, I and II, for $x > 0$ and $x < 0$, respectively, for which, by definition, $B(I) > 0$ and $B(II) < 0$.

Then, for the initial state (before the system enters the magnetic field) $|\Psi(t = 0)\rangle_{S+CM} = |\uparrow\rangle_{Sz} \otimes (|I\rangle_{CMx} + |II\rangle_{CMx})/\sqrt{2}$, one obtains:

$$|\Psi(t)\rangle_{S+CM} = \exp(-\imath t\hat{H}_{int}/\hbar)|\Psi(t = 0)\rangle_{S+CM} = (|\downarrow\rangle_{Sy}|I\rangle_{CMx} + |\uparrow\rangle_{Sy}|II\rangle_{CMx})/\sqrt{2}, \quad (3a)$$

where $|\cdot\rangle_{Si}$ represent the eigenstates of $\hat{S}_{Si}, i = x, y$ and the condition $\mu_B B'_\circ t = \pi/2$ (we set $|x| = 1$) is fulfilled; $t \leq \Delta t$, where Δt is the time interval in which the atom traverses the magnetic field.

Then the decoherence effect due to the environment R (cf. Sect. 2) gives for the final state of the system $S + CM$ the following mixed state:

$$\hat{\rho}_{S+CM} = \mathrm{tr}_R \hat{\rho}_{S+CM+R} = (|\downarrow\rangle_{Sy}|I\rangle_{CMxSy}\langle\downarrow|_{CMx}\langle I| + |\uparrow\rangle_{Sy}|II\rangle_{CMxSy}\langle\uparrow|_{CMx}\langle II|)/2, \quad (3b)$$

as formally presented by equation (2).

The expressions equations (3a, 3b) follow from the apparent analogy between a part of the set-up Figure 1 and the SG-experiment set-up. Actually: *the role*

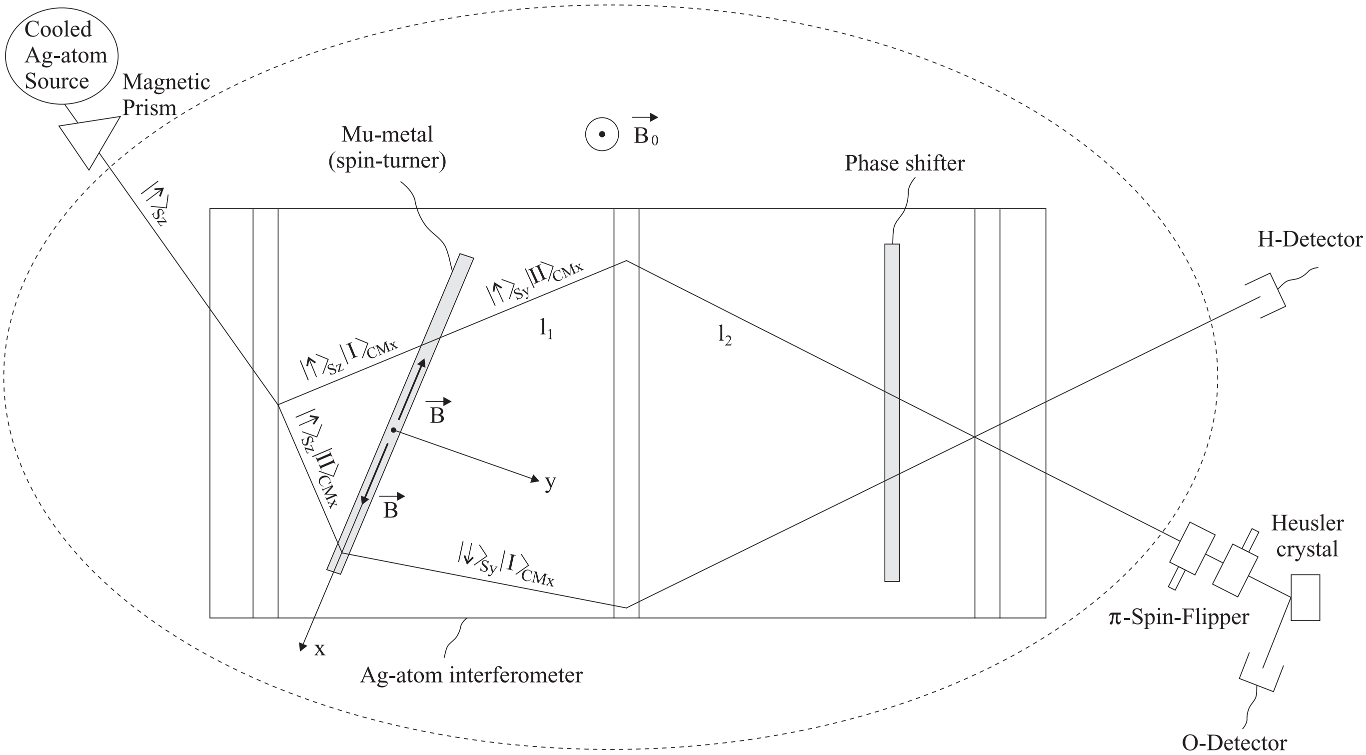

Fig. 1. A sketch of the experimental set-up: slightly changed set-up for the neutron optics interference of reference [6]. As distinct from the original set-up, our proposal set-up consists of the *properly cooled* beam of Ag atoms, as well as of the appropriate Ag-*atom interferometer*. The spin-turner (Mu-metal) magnetic field $\vec{B}$ and the space between the spin-turner and the phase shifter are analogous with the SG-magnet and the space between the SG-magnet and the screen in the SG experiment, respectively. The part of the set-up behind the spin-turner serves for the quantum measurements in analogy with the standard tests of the Bell's inequality. Due to the geometry in the spin-turner, the approximation for the spin-turner's magnetic field, $|B_x| = B'_\circ |x|$, is justified. The two "paths", I and II, are due to the passage of the atoms through the first plate of the interferometer. Behind the spin-turner, there are the correlations of states of the two "subsystems", S (spin) and CM (spatial "trajectories", I and II). Due to the decoherence effect (cf. Appendix A), these correlations are expected to be destroyed before the atom reach the phase shifter which results in non-violation of the Bell's inequality — cf. equations (8, 9).

of the SG magnet of Appendix A here is played by the beam-splitter and by the spin-turner (Mu-metal) magnetic field $\vec{B}$. The rest of the set-up Figure 1 behind the spin-turner serves as the "apparatus" for the quantum measurements necessary for testing the Bell's inequality. For the above choice (approximation) for the magnetic field and the condition $\mu_B B'_\circ t = \pi/2$, the self-dynamics due to $\hat{H}_{CM}$ (i.e. the wave packet spread — which is *not* accounted for in Eqs. (3a, 3b)) — can be neglected if the inequality for the standard deviation of the initial momentum $\Delta p^{(\circ)}_{CMx} \ll \pi/2$ is satisfied — cf. Appendix B. This condition can in principle be achieved by the *proper cooling* of the incidental beam of the atoms.

In general, the desired effect might be observable even if $\Delta p^{(\circ)}_{CMx} \not\ll \pi/2$ is satisfied, *if* the length, L, of the total "path" of the atoms in the set-up satisfies:

$$L \leq v_{min}\tau, \tag{4}$$

where $v_{min} \equiv v_{max} - \Delta v$, as defined in Appendix B, and τ is defined by equation (B.2) in Appendix B.

For the final state, equation (3b), one may easily prove non-violation of the Bell's inequality as follows. The task is to calculate the quantity S defined as:

$$S = E(\alpha_1, \chi_1) - E(\alpha_1, \chi_2) + E(\alpha_2, \chi_1) + E(\alpha_2, \chi_2), \tag{5}$$

where, *operationally*, $E(\alpha, \chi)$ is defined as:

$$E(\alpha,\chi) = \frac{N_{++}(\alpha,\chi)+N_{++}(\alpha+\pi,\chi+\pi)-N_{++}(\alpha+\pi,\chi)-N_{++}(\alpha,\chi+\pi)}{N_{++}(\alpha,\chi)+N_{++}(\alpha+\pi,\chi+\pi)+N_{++}(\alpha+\pi,\chi)+N_{++}(\alpha,\chi+\pi)}. \tag{6}$$

The probabilities $N_{++}(\alpha, \chi)$ are defined as:

$$N_{++}(\alpha,\chi) = \mathrm{tr}\hat{\rho}_{S+CM}\hat{P}^{(S)}_\alpha \otimes \hat{P}^{(CM)}_\chi, \tag{7}$$

for the quantum state $\hat{\rho}_{S+CM}$, while the quantum measurements for testing the Bell's inequality are defined by the following projectors:

$$\hat{P}^{(S)}_\alpha = \frac{1}{2}(|\downarrow\rangle_S + \exp(\imath\alpha)|\uparrow\rangle_S) \times ({}_S\langle\downarrow| + \exp(-\imath\alpha)\,{}_S\langle\uparrow|), \tag{7a}$$

$$\hat{P}^{(CM)}_\chi = \frac{1}{2}(|I\rangle_{CM} + \exp(\imath\chi)|II\rangle_{CM}) \times ({}_{CM}\langle I| + \exp(-\imath\chi)\,{}_{CM}\langle II|), \tag{7b}$$

for the system S and CM, respectively.

For the choices: $\alpha_1 = \pi/2$, $\alpha_2 = 0$, $\chi_1 = -\pi/4$ and $\chi_2 = \pi/4$, while the quantum state for the composite system reads $|\Psi\rangle = 2^{-1/2}(|\downarrow\rangle_S |I\rangle_{CM} + |\uparrow\rangle_S |II\rangle_{CM})$, one obtains [6]: $N_{++}(\alpha, \chi) = [1 + \cos(\alpha + \chi)]/4$, and therefore $E(\alpha, \chi) = \cos(\alpha + \chi)$, which finally gives rise to the violation of the Bell's inequality, $S = 2^{3/2} > 2$.

However, for the initial mixed state equation (3b) — where $|\cdot\rangle_S \equiv |\cdot\rangle_{Sy}$ and $|\cdot\rangle_{CM} \equiv |\cdot\rangle_{CMx}$ — the given choice of the parameters α, χ, gives for the probabilities $N_{++}(\alpha, \chi)$:

$$N_{++}(\alpha, \chi) = 1/4, \quad \forall \alpha, \chi, \tag{8}$$

i.e.

$$E(\alpha, \chi) = 0, \quad \forall \alpha, \chi, \tag{9}$$

which *gives rise to the validity of the Bell's inequality*, $S = 0$.

Therefore, our proposal for the experimental observation of the decoherence effect (non-violation of the Bell's inequality) reads: a properly cooled beam of the atoms (such that the initial spread in the momentum $\Delta p^{(\circ)}_{CMx} \ll \pi/2$) should be injected in the interference set-up as presented by Figure 1, while the condition:

$$\mu_B B'_\circ t = \pi/2, \tag{10}$$

is fulfilled. The observation of decoherence should be performed essentially as in the neutron optics experiment [6] through the quantum measurements of the observables defined as: $\hat{A}_S = \hat{P}^{(S)}_\alpha - \hat{P}^{(S)}_{\alpha+\pi}$ and $\hat{B}_{CM} = \hat{P}^{(CM)}_\chi - \hat{P}^{(CM)}_{\chi+\pi}$. If the condition $\Delta p^{(\circ)}_{CMx} \ll \pi/2$ can not be fulfilled, then the total length of the "path" of the injected atoms should satisfy the condition equation (4).

So, the desired effect should be observable in either choice of the *parameters of the experimental set-up*: (i) if the condition $\Delta p^{(\circ)}_{CMx} \ll \pi/2$ and equation (10) are satisfied, or (ii) if the former condition is not satisfied, then the total length of the "path" of the atom should satisfy (cf. Eqs. (4, B.2)) $L \leq (v_{max} - \Delta v)\hbar/m(\Delta v)^2$, where m is the mass and Δv is the standard deviation of the initial velocity of the Ag atom.

As distinct from SG experiment, the desired effect in our proposal *may fail* due to the following reasons. First, if there appear (e.g., in the interference set-up) some uncontrollable effects (e.g., the decoherence effect). Then, simply, our model would prove naive. Second, within the model used, the desired effect might fail to appear if the constraints on the set-up-parameters are not (or can not be) fulfilled. E.g., if the length $l_1 + l_2$ (cf. Fig. 1) is small enough that the atoms traversing this distance do not experience the decoherence effect — i.e. the decoherence effect is not completed. To this end, there is another *constraint on the set-up-parameters*: $l_1 + l_2 > (v_{max} - \Delta v) \cdot \tau_3$, where $\tau_3 \equiv \tau_D - \tau_2$, and the *"decoherence time"* τ_D should be estimated *in situ*, while the interval τ_2 is defined in Appendix A.

The conditions of the observability of the desired effect are summarized in Table 1.

Table 1. The conditions for observability of the desired effect, for the two "choices" for $\Delta p^{(\circ)}_{CMx}$ — the initial spread of the x-component of the atom's center-of-mass momentum. Notation: $B'_\circ$ — cf. Appendix A; v_{max}, Δv, m, represent the maximum and the spread in the initial velocity-distribution, and the mass of the atom, respectively; $\tau_3 \equiv \tau_D - \tau_2$, where τ_D is the so-called decoherence time, while τ_2 is the characteristic time for the entanglement formation in the $S + CM + R$ system.

$\Delta p^{(0)}_{CMx} \ll \pi/2$	$\Delta p^{(0)}_{CMx} \sim \pi/2$
$\mu_B B'_0 t = \pi/2$	$\mu_B B'_0 t = \pi/2$
$l_1 + l_2 \geq (\nu_{max} - \Delta\nu)\tau_3$	$l_1 + l_2 \geq (\nu_{max} - \Delta\nu)\tau_3$
	$L \leq \frac{(\nu_{max} - \Delta\nu)h}{2\pi m \left(\Delta p^{(0)}_{CMx}\right)^2}$

4 Preparation of the initial atomic beam

The model of the composite system $S + CM$ (cf. Eqs. (A.1–A.6) in Appendix A) that we employ is essentially attributable to Bohm [12]. Nevertheless, *this model is in the general use* in the research work devoted to the SG issues (cf., e.g., [14–16], and references therein). The theoretical studies employ the different methods of calculating the system's dynamics — cf., e.g., reference [14] in contrast to Bohm [12] — but the results are certainly mutually equivalent. Therefore, it seems that the only remaining "critical" point in our proposal refers to the stage of preparation of the initial atomic beam.

As to the initial state of the CM system, we pose the constraint (cf. Tab. 1) on $\Delta p^{(\circ)}_{CMx}$. To this end, we suggest a proper cooling of the atomic beam without entering the concrete experimental tasks in this regard.

As to the polarization of the spin — preparation of the system S — we emphasize a need for a specific, rather than for arbitrary polarization. Actually, for certain partially polarized atomic beams, there might appear nonvalidity of the Bell's inequality even without taking the environment R into account. Needless to say, in such cases the occurrence of decoherence — that should be revealed through non-violation of the Bell's inequality — is "masked". More precisely, as we show in Appendix C, for the mixed (partial) initial polarization:

$$\hat{\rho}_S = \omega_1 |\uparrow\rangle_{Sz\,Sz}\langle\uparrow| + \omega_2 |\downarrow\rangle_{Sz\,Sz}\langle\downarrow|, \tag{11}$$

as long as $\omega_1 > 2^{-1/2}$, one should expect violation of the Bell's inequality. In Appendix C, we also show that *taking the environment R into account* gives rise to non-violation of the Bell's inequality independently on the value of $\omega_1, 0 < \omega_1 < 1$. Therefore, for the sake of our program, the initial spin polarization $\hat{\rho}_S$ should be such that $\omega_1 > 2^{-1/2}$. Then, non-violation (validity) of the Bell's inequality should be ascribed *solely* to the decoherence effect as described in Section 3.

The efficient spin polarization methods exist (cf., e.g., [15,16], and references therein), including the full spin polarization [16], for which $\omega_1 = 1 > 2^{-1/2}$ (or $\omega_2 = 1$). In Figure 1, we assume the fully polarized spin along the $(+z)$-axis, albeit the polarization $\hat{\rho}_S$, for which $\omega_1 > 2^{-1/2}$, is *sufficient for our purposes*. Finally, we suggest in this regard the use of the Magnetic Prism Polarizer, which employs the Stern-Gerlach effect [6,16].

5 Discussion

Our analysis of the Stern-Gerlach experiment does not only reveal the subtlety of the quantum measurement of spin-1/2, but also offers the possibility to definitely answer the question "where (in this measurement) the effect of decoherence takes place?". Actually, it is sometimes assumed that only on the screen (which captures the incoming atoms) one has the final effect, and consequently, that there is not the decoherence effect before the screen. As opposite to this viewpoint, we give (cf. Sect. 2) the arguments for the occurrence of the decoherence effect *before the screen*. Then, one seems forced to conclude that the answer to the above question reads: the decoherence effect unfolds *before the atom reach the screen*, due to the effectiveness of the internal (and mesoscopic) environment — the system R in our notation — and should be considered to be *objective*, since no observer is present in between the source of the atoms and the screen.

Reducing the physical processes equations (3a, 3b) onto SG experiment of Appendix A is justified also by the following observations. In front of the spin-turner magnetic field ($\vec{B}$, in our notation), the Ag atoms give rise to the *diffraction effect* on the first plate of the interferometer, thus defining the two interfering "paths" I and II in the $(x-y)$-plane. This diffraction is possible due to ineffectiveness of the interaction $\hat{H}_{CM+R}$ relative to $\hat{H}_{CM}$, equation (A.4), as distinguished in Appendix B for the case that no external magnetic field is applied. Only in the domain of the spin-turner magnetic field, the effective interaction $\hat{H}_{S+CM+R} = \hat{H}_{S+CM} + \hat{H}_{CM+R}$ may dominate in the system, and if the condition $\Delta p^{(\circ)}_{CMx} \ll \pi/2$ is fulfilled, then the self-dynamics due to $\hat{H}_{CM}$ can be neglected (cf. Appendix B). Therefore, the two different effects are employed in the experiment proposed. First, there is the spatial interference of the Ag atom beam, and later (in between the spin turner and the spin shifter), the decoherence of the different, implicit "paths" I and II, takes place, just as it happens in the SG experiment.

The assumption of the coupling in the system $CM+R$ is essential for our considerations, for few reasons. First, it reveals "where" (in the spin-1/2 measurements) the decoherence takes place, thus making the effect of decoherence *objective* — no observer directly reads out the state of the system R (or of $S+CM$) — while allowing the *interpretation in terms of the single objects* (i.e. of the single atoms). Second, the condition $l_1+l_2 > v_{min}\tau_3$ (cf. Tab. 1) might not be fulfilled, in which case the *decoherence effect is simply not completed*, thus eventually giving rise to violation of the Bell's inequality. The "transition" between the two cases, $l_1+l_2 > v_{min}\tau_3$ and $l_1+l_2 < v_{min}\tau_3$, bears similarity with the effect of the "*short-living entangled states*", which has recently been experimentally observed [17], thus implicitly revealing the "border territory" between the "quantum" ($l_1+l_2 < v_{min}\tau_3$) and the "classical" ($l_1+l_2 > v_{min}\tau_3$) in the experiment here proposed. Third, and eventually the most interesting, the desired effect directly reveals the possibility of effectiveness of the *mesoscopic* yet *internal* environment for the occurrence of the decoherence effect. To this end, it is a fundamental question in the foundations of quantum physics generally, "where is a border line between the 'quantum' and 'classical' worlds?", or — as it is suggested by the recent experiments with the fullerene interferometry [18] — there is not a sharp line dividing the two physical worlds. Needless to say, sharpening this question might open a new path in the search for the microscopic origin of the phenomenological irreversibility, which is the central issue of the problem of the "transition from quantum to classical" [1,2]. Certainly, all these answers crucially depend on the outcome of the proposed experiment.

As to the experiment proposed, we assume that *no other mechanism of decoherence of the different paths for the Ag atoms beam proves effective*. In this regard, we resort to the *generally adopted [12,14–16] model of SG experiment* which does not presume any other mechanism of decoherence. Even more important, we assume that the presumed mechanism of decoherence is universal for the Ag atoms, by simply not having the reason to doubt about the absence of this effect if — as it seems necessary — it reveals itself in the spin-1/2 measurement. So, the two main possibilities concerning the outcomes of the experiment proposed are the following. *First*, if the experiment does not reveal non-validity of the Bell's inequality, we meet the necessity of the substantial revision of the foundations of the quantum measurement theory (e.g., then, the decoherence takes place on the screen, and we do not have the job with the "retrospective" measurement (of the "second kind") on the screen — contrary to the common wisdom [7,9,19]). *Second*, in the case the experiment reveals validity of the Bell's inequality, we end up with the consistent physical picture with the nontrivial observations as essentially distinguished above. To this end, it is worth emphasizing that (supposed) effectiveness of the internal, mesoscopic environment justifies importance of the investigation of the internal environment in the "macroscopic" world (i.e. in the quantum physics of complex, many-particle systems). Finally, this would be the first observation ever of the influence of the internal environment, which is of the order of 10^2 particles, so extending the standard wisdom about the necessity of the "macroscopic environment" for the occurrence of the decoherence effect [1,2,10,20].

6 Conclusion

In certain experimental situations analogous to the standard Stern-Gerlach experiment, one may expect the occurrence of the decoherence effect due to the *mesoscopic, internal* environment. With the slight (not operationally trivial yet) changes of the set-up for the neutron optics interference, we are able to distinguish the range for the set-up-parameters that might allow the occurrence of decoherence and consequently non-violation of the Bell's inequality for certain (spatial) degrees of freedom of a *single atom*, in contradistinction to the similar experiments with neutrons.

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Appendix A

The composite system for the Stern-Gerlach experiment (measurement of the spin-1/2) is defined as "(atom's) spin + center-of-mass + relative coordinates + SG-magnet + the field" — $S + CM + R + M + F$. That is, *a single* Ag *atom* is considered as *a composite system* defined as $S + CM + R$.

Prima facie, the Hamiltonian of the system is defined as:

$$\hat{H} = \hat{H}_S + \hat{H}_{CM} + \hat{H}_R + \hat{H}_{S+CM}, \quad \text{(A.1)}$$

where $\hat{H}_S$ and $\hat{H}_R$ represent the Hamiltonians of the non-interacting "systems", where $\hat{H}_{S+CM}$ is the interaction Hamiltonian for the composite system $S + CM$ due to the applied external magnetic field $\vec{B}(\hat{x}_{CM})$; x_{CM} is the x-coordinate of the center-of-mass of the atom. Needless to say, $\hat{H}_{S+CM} = \mu_B \hat{S}_{Sx} \otimes B_x(\hat{x}_{CM})$; $\mu_B \equiv |e|\hbar/2mc$. It is *essential to emphasize* that, *in this model*, the SG magnet plays the role of the field source, and not of a dynamical system. That is, in the standard (and *generally used*) model (cf., e.g., [12,14–16]), *neither the field, nor the SG magnet itself are subject to any changes due to the interaction with the atom*. So, the variables of the SG magnet and of the field appear just as the *parameters*, through the magnetic field, $\vec{B}$, which can be modeled as [16]:

$$\vec{B} = (B_x, 0, 0), \quad B_x = B'_\circ \hat{x}_{CM}. \quad \text{(A.2)}$$

Therefore, one may conclude that, in the given model, neither SG magnet, nor the magnetic field might play the role of the "apparatus" for the spin-measurement. Therefore, the model Hamiltonian (A.1) should be redefined in order to include the "apparatus", which should provide the successful measurement.

In the composite system $S+CM+R$, *only* the subsystem R might play such a role. Actually, with the formal introduction of the interaction Hamiltonian $\hat{H}_{CM+R}$, one can make the whole picture consistent. That is, with a proper modeling of $\hat{H}_{CM+R}$, one may formally obtain the conditions necessary for the occurrence of the decoherence effect concerning the system CM [11]. Furthermore, due to the correlations of states in the system $S + CM$ — which, in turn, are due to $\hat{H}_{S+CM}$ — the decoherence effect also refers to the composite system $S + CM$.

The simplest model for $\hat{H}_{CM+R}$ utilizing the desired conditions reads (cf. Dugić [11] for technical details):

$$\hat{H}_{CM+R} = C(|I\rangle_{CMCM}\langle I| - |II\rangle_{CMCM}\langle II|) \otimes \hat{D}_R, \quad \text{(A.3)}$$

where C is the coupling constant, and $\hat{D}_R$ is arbitrary observable of the system R; we denote $|I\rangle\langle I| \equiv \int_0^\infty |x\rangle dx \langle x|$, and $|II\rangle\langle II| = \hat{I}_x - |I\rangle\langle I|$. A straightforward generalization of this model allows accounting for the possibility of measurement of arbitrary projection of the spin, $\hat{S}_n$ ($\vec{n}$ is a unit vector determining the axis, i.e. the spin projection).

Now, if the inequalities:

$$\|\hat{H}_{S+CM}\| \gg \|\hat{H}_{CM}\|, \quad \|\hat{H}_{CM+R}\| \sim \|\hat{H}_{CM}\|, \quad \text{(A.4)}$$

are fulfilled (the effective interaction in the system $(S + CM) + R$ is $\hat{H}_{S+CM+R} = \hat{H}_{S+CM} + \hat{H}_{CM+R}$), then it is straightforward [11] to prove that the composite system's dynamics is given by the following state transformations:

$$\sum_i C_i |i\rangle_S |0\rangle_{CM} |0\rangle_R \xrightarrow{\tau_1} (C_1|\downarrow\rangle_S |I\rangle_{CM} + C_2|\uparrow\rangle_S |II\rangle_{CM})|0\rangle_R \xrightarrow{\tau_2'} C_1|\downarrow\rangle_S |I\rangle_{CM} |+\rangle_R + C_2|\uparrow\rangle_S |II\rangle_{CM} |-\rangle_R, \quad \text{(A.5)}$$

where the spin states refer to the x-projection of the spin, and τ_1, $\tau_2 \equiv \tau_1 + \tau_2'$, are the characteristic time intervals for the entanglement formation in the system $S+CM$ and $(S + CM) + R$, respectively [21].

Now, the tracing out the environmental (R's) degrees of freedom, "tr_R", gives for the $S+CM$'s (reduced) density matrix:

$$\hat{\rho}_{S+CM} = |C_1|^2 |\downarrow\rangle_S |I\rangle_{CMS}\langle\downarrow|_{CM}\langle I| + |C_2|^2 |\uparrow\rangle_S |II\rangle_{CMS}\langle\uparrow|_{CM}\langle II|, \quad \text{(A.6)}$$

in accordance with the SG-experiment effect, equation (2).

The three dynamical processes dominate in the SG experiment as well as in the set-up Figure 1: (i) the wave packet splitting (i.e. the "path" (I and II) interference) due to the self-Hamiltonian $\hat{H}_{CM}$ (the characteristic time τ discussed in Appendix B), (ii) the entanglement formation due to $\hat{H}_{S+CM}$ (the characteristic time τ_1), and (iii) the entanglement formation due to the interaction $\hat{H}_{CM+R}$ (the characteristic time τ_2). While the processes (i) and (iii) are present all the time of the experiment, the process (ii) takes place only in the spatial area of the magnetic field.

So, as regards Figure 1, the three processes are simultaneous in the spatial area of the spin-turner magnetic field. Now, given the plausible assumptions equation (A.4), the ordering in equation (A.5) — which are due to the interactions $\hat{H}_{S+CM}$ and $\hat{H}_{CM+R}$, respectively — can be easily proved [11], while the decoherence effect equation (A.6) *should be completed (effective) before the atom reach the phase shifter.* To this end, the *decoherence time*, τ_D, not explicit in equation (A.5), remains undetermined, except that it is expectable to satisfy the inequality $\tau_D \gg \tau_2$ [21]. That is, by τ_2 we denote the lower bound [21] on the time necessary for the entanglement formation in the $CM + R$ system. If τ_2 is the characteristic time for this process, then the estimation $\tau_D \gg \tau_2$ ($\tau_D = \tau_2 + \tau_3$) — cf. reference [21], footnote 2 — stems the unfolding of the decoherence effect also behind the spin-turner set-up.

The dynamical process (i) is the subject of Appendix B.

Appendix B

The strength of the external magnetic field $\vec{B}(\hat{x}_{CM})$ is essential for both, equation (A.4) to be fulfilled, as well as for the appearance of the desired effect. As to the later, it is well-known [12], that the product $\mu_B B'_\circ t$ should be much greater than the spread in the initial momentum along the x-direction, $\Delta p^{(\circ)}_{CMx}$, in order to *provide distinguishability (nonintersecting) of the two spots on the screen.* That is, for the magnetic field as defined by equation (A.2), the condition (cf. Eq. (10), where $\mu_B B'_\circ t = \pi/2$):

$$\pi/2 = \mu_B B'_\circ t \gg \Delta p^{(\circ)}_{CMx}, \tag{B.1}$$

guarantees the well-defined (nonintersecting) spots on the screen; then the dynamics due to $\hat{H}_{CM}$ (i.e. the wave packet spread–the process (i) in Appendix A) *can be neglected* [12].

To this end, one may wonder if the path of an atom in free space (i.e. when *no external field is applied*) are well defined due to the interaction $\hat{H}_{CM+R}$. However, as it is well-known from the general decoherence theory [11], only for a sufficiently strong interaction $\hat{H}_{CM+R}$ this effect might be unavoidable — which is not the case in the model studied, cf. equation (A.4). So, the role of the *strong* external magnetic field is threefold. First, it provides splitting of the two "paths" (I and II) for the atom [12]. Second — cf. equation (B.1) — the strong field guarantees nonintersection (i.e. mutual distinguishability) of the spots on the screen [12]. Third, *only* for the strong magnetic field, the effective interaction $\hat{H}_{S+CM+R}$ ($\hat{H}_{S+CM+R} = \hat{H}_{S+CM} + \hat{H}_{CM+R}$) is *strong enough* to provide (cf. Eq. (A.4)) the occurrence of the decoherence effect, equation (A.6). The decoherence process starts being efficient in the magnetic field spatial area, and completes in between the SG-magnet and the screen (cf. Appendix A). As to Figure 1, the *decoherence effect should be completed (effective) in between the spin-turner and the phase shifter.*

However, if the condition equation (B.1) can not be obeyed (e.g. for some practical reasons), then the distinguishability of the spots on the screen can be achieved in the time intervals less than the following interval τ:

$$\tau = m\hbar/(\Delta p^{(\circ)}_{CMx})^2, \tag{B.2}$$

which easily follows (for our case) from the general expression ($\tau = [(\Delta p)^2 d^2\omega\ /dp^2]^{-1/2}$) for the wave packet spread [12]. Therefore, virtually independently on the strength of the external magnetic field — *except for the conditions* equation (A.4) to be fulfilled — for the time intervals of the order less than τ given by equation (B.2), one would have perfect distinguishability of the desired spots on the screen. As a constraint, one should design the experiment so that the total length of the "flight" of the atoms should not exceed the length $v_{min}\tau$, where $v_{min} \equiv v_{max} - \Delta v$, and v_{max} is the maximum, while Δv is the standard deviation, of the atom's velocity distribution along the x-direction.

Appendix C

Let us assume the partial initial polarization of the spin:

$$\hat{\rho}_S = \omega_1 | \uparrow \rangle_{SzSz} \langle \uparrow | + \omega_2 | \downarrow \rangle_{SzSz} \langle \downarrow |. \tag{C.1}$$

In analogy with equation (3a), *neglecting the environment R*, one obtains for the final state of the $S + CM$ system as follows:

$$\hat{\rho}_{S+CM} = \omega_1 |\Psi_1\rangle\langle\Psi_1| + \omega_2 |\Psi_2\rangle\langle\Psi_2|, \tag{C.2}$$

where

$$|\Psi_1\rangle = 2^{-1/2}(| \downarrow\rangle_{Sy}|I\rangle_{CMx} + | \uparrow\rangle_{Sy}|II\rangle_{CMx}), \tag{C.3}$$

$$|\Psi_2\rangle = 2^{-1/2}(| \uparrow\rangle_{Sy}|I\rangle_{CMx} + | \downarrow\rangle_{Sy}|II\rangle_{CMx}). \tag{C.4}$$

Now, the probabilities defined in equation (7) read:

$$N_{++}(\alpha, \chi) = \mathrm{tr}\hat{\rho}_{S+CM} \hat{P}^{(s)}_{(\alpha)} \otimes \hat{P}^{(p)}_{(\chi)}, \tag{C.5}$$

which, after the simple algebra and for the choices of α's and χ's as in Section 3, gives:

$$E(\alpha_1, \chi_1) = -E(\alpha_1, \chi_2) = -2^{-1/2} + \omega_1 2^{1/2}, \tag{C.6}$$

$$E(\alpha_2, \chi_1) = E(\alpha_2, \chi_2) = 2^{1/2}. \tag{C.7}$$

Finally,

$$S = E(\alpha_1, \chi_1) - E(\alpha_1, \chi_2) + E(\alpha_2, \chi_1) + E(\alpha_2, \chi_2) = 2^{3/2}\omega_1. \tag{C.8}$$

Obviously, for $\omega > 2^{1/2}$, there occurs the violation of the Bell's inequality, while for $\omega \leq 2^{1/2}$, the Bell's inequality is satisfied.

Taking the environment R into account, for the initial state equation (C.1), one obtains:

$$N_{++}(\alpha, \chi) = 1/4,\ \forall \alpha, \chi, \tag{C.9}$$

thus giving rise to $E(\alpha, \chi) = 0$, $\forall \alpha, \chi$, i.e. to the *validity of the Bell's inequality* — $S = 0$.

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